Operators

① Position	② Momentum	③ Angular Momentum
$\hat{x} = x$	$\hat{P}_x = -i\hbar \frac{\partial}{\partial x}$	$\hat{L}_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$
$\hat{y} = y$	$\hat{P}_y = -i\hbar \frac{\partial}{\partial y}$	$\hat{L}_y = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$
$\hat{z} = z$	$\hat{P}_z = -i\hbar \frac{\partial}{\partial z}$	$\hat{L}_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$

Properties of operators

1	$\hat{A} f(x) =$ operator operates one time
2	$\hat{A}^2 f(x) =$ —— // —— two ——
3	$\hat{A}^3 f(x) =$ —— // —— three ——
4	Commutator $[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} = 0$ then $\hat{A}$, $\hat{B}$ are Commute
5	Every operator commute with itself
6	Multiplication operator commute with each other
7	Anticommutator $\{\hat{A}, \hat{B}\} = \hat{A}\hat{B} + \hat{B}\hat{A}$

Some important relations

① Position: [Position, Position] $= 0$

$[x, y] = 0$

$[y, z] = 0$

$[y^2, x] = 0$

$[x, x] = 0$

ie Commutator of Position with any Position is zero

② Linear Momentum

$[x, p_x] = i\hbar$

$[y, p_y] = i\hbar$

$[z, p_z] = i\hbar$

$[x, p_x^n] = n i\hbar\, p_x^{n-1}$

$[y, p_y^n] = n i\hbar\, p_y^{n-1}$

$[z, p_z^n] = n i\hbar\, p_z^{n-1}$

ie Commutator of Position with momentum in same direction is $\neq 0$

$[x^n, p_y] = 0$

$[x^n, p_z] = 0$

$[y^n, p_x] = 0$

$[y^n, p_z] = 0$

$[z^n, p_x] = 0$

$[z^n, p_y] = 0$

ie commutator of Position with momentum in different direction $= 0$ for any value of n this relation is true

$[P_x, P_y] = 0$

$[P_x, P_z] = 0$

$[P_x, P_x^2] = 0$

$$\left[x^n, \frac{d}{dx}\right] = -n x^{n-1}$$

$$[\hat{H}, x] = \frac{-\hbar^2}{m} \frac{\partial}{\partial x}$$

$[P_x, x^2] = -i\hbar\, 2x$

$$[P_y, y^3] = \frac{\hbar}{i} 3y^2$$

Conclusion :.

$$\boxed{[P_x, x^n] = -n i\hbar\, x^{n-1}}$$

lly for

$[P_y, y^n] =$

&

$[P_z, z^n] =$

$$[x^3, P_x] = \frac{3x^2 h i}{2\pi}$$

$$[x^4, P_x] = \frac{4x^3 h i}{2\pi}$$

$$[z^4, P_z] = \frac{4z^3 h i}{2\pi}$$

Conclusion

$$[x^n, P_x] = n x^{n-1} i\hbar = \frac{n x^{n-1} i h}{2\pi}$$

lly for $[y^n, P_y]$ & $[z^n, P_z]$

$$[L_x, L_y] = i\hbar L_z$$

$$[L_y, L_z] = i\hbar L_x$$

$$[L_z, L_x] = i\hbar L_y$$

$x \to y \to z \to x$ +ve

$$[L_y, L_x] = -i\hbar L_z$$

$$[L_z, L_y] = -i\hbar L_x$$

$$[L_x, L_z] = -i\hbar L_y$$

$x \leftarrow y \leftarrow z \leftarrow x$ −ve

$$[L_x, y] = i\hbar z$$

$$[L_y, z] = i\hbar x$$

$$[L_z, x] = i\hbar y$$

$x \to y \to z \to x$

$$[L_x, x] = 0$$

$$[L_y, y] = 0$$

$$[L_z, z] = 0$$

$$[L_x^2, L_x] = 0$$

ie

$$[L_x^n, L_x] = 0$$

$$[L_y^2, L_y] = 0$$ ie

$$[L_y^n, L_y] = 0$$

$$[L_z^2, L_z] = 0$$

$$[L_z^n, L_z] = 0$$

Ladder operator

① Raising operator / ladder up

$$L_+ = L_x + iL_y$$

② Lowering operator / Ladder down

$$L_- = L_x - iL_y$$

$$[L_z, L_+] = \hbar L_+$$

$$[L_z, L_-] = -\hbar L_-$$

$$[L_+, L_-] = 2\hbar L_z$$

orthonormal

$$\int \psi_m^* \psi_m \, d\tau = 1$$

orthogonal

$$\int \psi_m^* \psi_n \, d\tau = 0$$

For ple in a box

① Energy is additive property.

$$E_{total} = E_x + E_y + E_z$$

② wave fun is multiplicative

$$\psi_{total} = \psi_x \cdot \psi_y \cdot \psi_z$$

For ple in a box

Average value is: $\langle x \rangle = \frac{l}{2}$

$$\langle x^2 \rangle = \frac{l^2}{3}$$

$$\langle x^3 \rangle = \frac{l^3}{4}$$

$$\langle x^n \rangle = \frac{l^n}{n+1}$$

Average momentum of ple in a box is $\langle p_x \rangle = 0$