Section-I Previous Year Problems

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8 g of N_2 is expanded reversibly from 1 lit to 10 lit at 300 K. Calculate ΔS, ΔH and ΔG (At. mass N = 14).

Given Weight = 8 gm N_2

Molecular weight of $N_2 = 2 \times N = 2 \times 14 = 28$ gm/mol

$$(n) \text{ no. of moles of } N_2 = \frac{\text{given weight}}{\text{Molecular weight}} = \frac{8 \cancel{gm}}{28 \cancel{gm}/mol}$$

$= 0.2857$ moles

V_1 = Initial vol = 1 Lit

V_2 = final vol = 10 Lit

$T = 300$ K

formula for ΔS

$$S = \frac{q}{T}$$

from 1st law

$$U = q + w$$

$$\Delta U = q + (-P_{ext}\, dV)$$

at constant temp

$$\Delta U = 0$$

$$\therefore q = P_{ext}\, dV$$

from ideal gas eqn

$$P_{ext} = \frac{nRT}{V}$$

$$\therefore q = nRT \frac{dV}{V}$$

formula for ΔG from maxwell eqn

$$dG = V dp - S dT$$

at constant temp.

$$dT = 0$$

$$\therefore dG = V dp$$

from ideal gas eqn

$$V = \frac{nRT}{P}$$

$$\therefore dG = \frac{nRT}{P} dp$$

$$\int_{G_1}^{G_2} dG = nRT \int_{P_1}^{P_2} \frac{dp}{P}$$

$$\Delta G = nRT \ln\left(\frac{P_2}{P_1}\right)$$

$\therefore q = nRT\frac{dv}{V}$

$\therefore S = \frac{q}{T}$

$S = \frac{nR\cancel{T}\frac{dv}{V}}{\cancel{T}}$

$\int_{S_1}^{S_2} S = nR\int_{V_1}^{V_2}\frac{dv}{V}$

$$\boxed{\Delta S = nR\ln\frac{V_2}{V_1}}$$

$\Delta S = \underset{\substack{\uparrow\\ \text{unit}\\ \cancel{\text{mol}}}}{0.2857} \times \underset{\substack{\uparrow\\ \text{unit}\\ J\,K^{-1}\,\cancel{mol}^{-1}}}{8.314} \times \ln\left(\frac{10}{1}\right)$ $\uparrow \frac{\cancel{\text{Lit}}}{\cancel{\text{Lit}}}$

$$\boxed{\Delta S = 5.494\ J\,K^{-1}}$$

Comment = ΔS +ve spontuneous

$\Delta G = nRT\ln\left(\frac{}{P_1}\right)$

as $PV = nRT$

$P_1 = \frac{nRT}{V_1}$

& $P_2 = \frac{nRT}{V_2}$

At constant temp.

Put P_1 & P_2 in ΔG

$\therefore \Delta G = nRT\ln\left(\frac{n\cancel{RT}/V_2}{n\cancel{RT}/V_1}\right)$

$$\therefore \boxed{\Delta G = nRT\ln\left(\frac{V_1}{V_2}\right)}$$

$\therefore \Delta G = \underset{\substack{\downarrow\\ \cancel{\text{mol}}}}{0.2857} \times \underset{\substack{\downarrow\\ J\,\cancel{K}^{-1}\,\cancel{mol}^{-1}}}{8.314} \times \ln\left(\frac{1}{10}\right) \times 300\,\cancel{K}$ $\frac{\cancel{\text{Lit}}}{\cancel{\text{Lit}}}$

$$\boxed{\Delta G = -1640.80\ J}$$

Comment = ΔG = −ve spontuneous

Now for ΔH

$\Delta G = \Delta H - \underset{\substack{\uparrow\\ 300\,K}}{T}\Delta S$

$\therefore \Delta H = \Delta G + T\Delta S$

$= -1640.80\,J + 300\,\cancel{K} \times 5.494\ J\,\cancel{K}^{-1}$

$$\boxed{\Delta H = 7.394\ J}$$

Three moles of hydrogen are compressed isothermally and reversibly from 60 dm³ to 20 dm³ and 8.22 KJ of work is done on it. Assuming ideal behaviour, calculate the temperature of the gas.

n = no. of moles = 3

$V_1 = 60\ dm^3$ for compression $V_2 < V_1$

$V_2 = 20\ dm^3$

$W = 8.22\ KJ = 8.22 \times 10^3\ J$

Formula; reversible Process

$$W = -P_{ext}\, dv$$

from ideal gas eqⁿ $P_{ext} = \frac{nRT}{V}$

$$\therefore W = -nRT \frac{dv}{V}$$

$$W = -nRT \int_{V_1}^{V_2} \frac{dv}{V}$$

$$W = -nRT \ln \frac{V_2}{V_1}$$

$$\therefore T = \frac{W}{-nR \ln\left(\frac{V_2}{V_1}\right)} = \frac{8.22 \times 10^3\ \cancel{J}}{-3 \times 8.314\ \cancel{J}\, K^{-1} \times \ln\left(\frac{20\ \cancel{dm^3}}{60\ \cancel{dm^3}}\right)}$$

$$\boxed{T = 299.98\ K} \approx 300\ K$$

Calculate the work done during adiabatic reversible expansion of 0.02mol [illegible]r at 25°c expanded from 0.5 L to 1.0 L.

[Given : $C_{v,m}$ of Ar = 12.48J K^{-1} mol^{-1}]

$V_1 = 0.5$ L

$V_2 = 1.0$ L

$T = 25°c = 298K$

$C_{v,m} = 12.48\ J\ K^{-1}\ mol^{-1}$ ie $n =$ 1 mole

$\text{formula} \xrightarrow{} .$ $\quad \Delta U = \underset{\uparrow}{q} + w$

$q = 0$ for adiabatic process

$\therefore \Delta U = W$

$W = -P_{ext}\, dV$ $\qquad \left(P_{ext} = \frac{nRT}{V} \right.$

$W = -nRT \int_{V_1}^{V_2} \frac{dV}{V}$

$\therefore W = -nRT \ln\left(\frac{V_2}{V_1}\right)$

$\therefore W = -0.02\,\cancel{mol} \times 8.314\, J\, \cancel{K^{-1}}\, \cancel{mol^{-1}} \times 298\cancel{K} \times \ln\left(\frac{1}{0.5}\right)$

$$\boxed{W = -34.34\ J}$$

Calculate ΔS, ΔG and ΔH when 20g of methane is mixed with 30g of ethane considering both of them to be ideal at 30°c. comment on your answer.

Formula.

$\Delta S_{mix} = -nR \sum x_i \ln x_i = -nR\left(x_A \ln x_A + x_B \ln x_B\right)$

$\Delta G_{mix} = nRT \sum x_i \ln x_i = nRT\left(x_A \ln x_A + x_B \ln x_B\right)$

$\Delta G_{mix} = nRT \sum x_i \ln x_i = nRT(x_A \ln x_A + x_B \ln x_B)$

n = Total no. of moles

x = mole fraction.

x_A = mole fraction of methane

x_B = — | ——— ethane

Now first Calculate mole fraction x_A & x_B & total no of moles

given weight of methane = 20 gm

Molecular weight of methane = $CH_4 = 12 + (1 \times 4) = 16 \text{ gm mol}^{-1}$

$$\text{no. of moles of methane} = n_A = \frac{\text{given Weight}}{\text{Molecular weight}} = \frac{20}{16} = \boxed{1.25 = n_A}$$

given weight of ethane = 30 gm

$$\text{Molecular} \text{ — | ———} = CH_3-CH_3 = (2\times12)+(1\times6) \longrightarrow = 24+6 = 30 \text{ gm mol}^{-1}$$

$$\text{no. of moles of ethane} = n_B = \frac{\text{given weight}}{\text{M.wt.}} = \frac{30 \cancel{\text{gm}}}{30 \cancel{\text{gm}} \text{ mol}^{-1}} = 1 \text{ mol}$$

$$\therefore x_A = \frac{n_A}{n_A+n_B} = \frac{1.25}{1.25+1.0} = 0.55$$

$$x_B = \frac{n_B}{n_A+n_B} = \frac{1.0}{1.25+1.0} = 0.44$$

n = Total no. of moles = $n_A + n_B = 1 + 1.25 = 2.25$

$$\therefore \Delta S_{mix} = -nR(x_A \ln x_A + x_B \ln x_B)$$

$$= -2.25 \cancel{\text{mol}} \times 8.314 \text{ J K}^{-1} \cancel{\text{mol}^{-1}} (0.55 \ln(0.55) + 0.44 \ln(0.44))$$

$= -2.25\,\cancel{\text{mol}} \times 8.314\ \text{J}\ \text{K}^{-1}\ \cancel{\text{mol}}^{-1}\ (0.55\ln(0.55) +$

$0.44\ln(0.44))$

$$\therefore \Delta S_{mix} = -18.70\ \text{J}\ \text{K}^{-1} \times \left(-0.3288 + (-0.3612)\right)$$

$$\boxed{\Delta S_{mix} = 12.90\ \text{J}\ \text{K}^{-1}}$$

Ily $\Delta G_{mix} = nRT\left(x_A \ln x_A + x_B \ln x_B\right)$

$$= \underset{\text{mol}}{\underset{\downarrow}{2.25}} \times \underset{\text{J K}^{-1}\text{mol}^{-1}}{\underset{\downarrow}{8.314}} \times \underset{\text{K}}{\underset{\downarrow}{(30+273)\text{K}}} \times \left(0.55\ln(0.55) + 0.44\ln(0.44)\right)$$

$$\boxed{\Delta G_{mix} = -3911.20\ \text{J}}$$

$$\Delta G_{mix} = \Delta H_{mix} - T\Delta S_{mix}$$

$$\therefore \Delta H_{mix} = \Delta G_{mix} + T\Delta S$$

$$= -3911.20 + 303\text{K} \times 12.90$$

$$\boxed{\Delta H_{mix} = 0}$$