Indian Knowledge System – Mathematics

Chapter 1 : Introduction

1. Multiple Choice Questions

(a) Which of the following texts is considered the earliest source of Indian mathematics?

Ans: Sulba Sūtras

(b) Who among the following used a binary number system in prosody?

Ans: Pingala

(c) Which ancient Indian number name corresponds to 1012?

Ans: Parārdha

(d) In the Katapayādi system, the syllables “cha” and “na” correspond to which digits?

Ans: 6 and 0

2. True/False Questions

(a) Bhāskara II was the first to assign rules for working with zero.

False

(b) The Katapayādi system reads numbers from right to left in its later usage.

False

(c) The Bhūtasankhyā system uses meaningless syllables to represent numbers.

False

(d) The decimal place-value system and zero were developed in India by 5th century

True

3. Short Answer Questions

(a) What is the meaning of the verse ”Yathā śikhā mayūrāṇām...” and what does it signify?

Meaning: "Like the crests on the heads of peacocks, like the gems on the hoods of the cobras, mathematics (astronomy) is at the top of the Vedanga śāstras."

Significance: This verse from Vedanga Jyotisa signifies the supreme importance and esteemed position of mathematics and astronomy among the branches of secular knowledge in ancient India.

(b) Write any four Sanskrit number names for numbers from 1 to 9.

* Eka (एक) - 1

* Dvi (द्वि) - 2

* Tri (त्रि) - 3

(चतुः) - 4

(Other possible answers include Panca, Sat, Sapta, Asta, Nava)

(c) What is the main principle of the Bhūtasankhyā number system?

The main principle of the Bhūtasankhyā system is that numbers are represented using meaningful words related to objects, natural elements, or ideas, which are placed in the same order as digits in the decimal system. These words are often written and read from right to left.

(d) Mention two mathematicians from the Kerala School of Mathematics and their contributions.

Madhava of Sangamagrama (around 1350 CE): Known for starting the Kerala School of Mathematics and developing infinite series for pi, sine, and cosine, which laid the foundation for concepts that later became part of calculus.

Nilakantha Somayaji (15th-16th centuries CE): A mathematician from the Kerala School who improved earlier ideas and wrote commentaries on astronomy and mathematics, including explaining why Aryabhata called the value of pi 'asananh' or approximate (irrational).

(Other possible answers include Parameshvara and Jyeshthadeva.)

4. Medium Answer Questions

(a) Explain the role of the Sulba Sūtras in early Indian mathematics.

The Sulba Sutras, dating from around 800 CE - 200 CE, are among the earliest mathematical texts in India and are part of the Vedic literature. They played a crucial role by providing rules for constructing Vedic altars using ropes, thereby incorporating geometrical ideas. These texts clearly present the Pythagorean Theorem long before its known appearance in Greek mathematics. They also discuss methods for squaring the circle and provide approximate values for irrational numbers like $\sqrt{2}$ and $\sqrt{3}$, demonstrating the early development of mathematical thinking in India.

(b) Describe the structure and use of the Katapayādi system with one example.

The Katapayādi system is an ancient Indian method for representing numbers using letters, which facilitates memorization of large numbers by converting them into words or verses. In this system, specific consonants are assigned numerical values from 0 to 9, arranged into four groups starting with 'ka', 'ta', 'pa', and 'ya'. Vowels have no numerical value and are used solely to form pronounceable words. While early versions read numbers from right to left, later versions, like modern numbering, read from left to right. This system was particularly prevalent in South India for astronomy, mathematics, and poetry.

Example: The value of π to 17 decimal places is encoded in the verse: "भद्राम्बुधिगसिद्धजन्मगणितश्रद्धा स्म यद् भूपगीः"

The relevant consonants and their assigned digits are:

भ (4) द्रा (2) म्बु (3) धि (9) ग (7) सि (9) द्ध (8) ज (5) न्म (3) ग (5) णि (6) त (2) श्र (9) द्धा (5) स्म (1) य (4) द् (1) भू (3) प (1) गीः (3)

Reading these digits in reverse order gives the value of π as 3.14159265358979324.

(c) What is the significance of mathematics in the Indian tradition, according to ancient scholars?

According to ancient scholars, mathematics held a position of supreme importance and was considered both a practical and a pure subject. It was initially developed as part of astronomy and religious literature, and later evolved to understand the world, solve practical problems, and support disciplines like architecture, music, and logic. The verse from Vedanga Jyotisa, "Like the crests on the heads of peacocks, like the gems on the hoods of the cobras, mathematics (astronomy) is at the top of the Vedanga śāstras," highlights its elevated status. Indian mathematics developed independently, contributing concepts like zero and the decimal place-value system. It also fostered clear thinking, logic, and constructive proof techniques useful for algorithms. Furthermore, Indian mathematical texts were translated into Arabic and Latin, facilitating the transmission of these ideas globally. Mahavira emphasized that mathematics is useful and necessary in all areas of life, from daily matters and Vedic rituals to religious duties, love, trade, music, and drama.

5. Long Answer Questions

(a) Discuss the major contributions of ancient Indian mathematicians to arithmetic, algebra, and astronomy with appropriate examples.

Ancient Indian mathematicians made remarkable contributions to arithmetic, algebra, and astronomy.

In Arithmetic, one of the greatest achievements was the invention of the decimal place-value system and the use of zero as a number. This foundationally changed counting and calculation methods worldwide. The concept of zero was treated as a number and used in calculations by Brahmagupta (598 CE). Basic operations like addition, subtraction, multiplication, and division (Parikarma) were well-established. The "rule of three" (Trairāśi) was also developed for practical computation. Early texts like the Bakhshali manuscript (200 CE) show advanced calculation methods, including those for fractions and square roots. Śrīdhara (around 750 CE) and Bhaskara II (1150 CE) further elaborated on number names and place value.

For Algebra, India's contributions were foundational, particularly the development of Avyaktaganita (algebra). Brahmagupta (598 CE) gave rules for working with negative numbers and methods to solve quadratic and indeterminate equations, which are still taught today. The Kuttaka method was developed for solving indeterminate equations. Bhaskara II (1114 CE) further advanced algebraic problems in his work Bijaganita, building upon the use of zero and negative numbers. He is also known for developing the Chakravala method, a powerful technique to solve Pell's equation ($Nx^2 + 1 = y^2$), a significant achievement in number theory. Simple algebraic equations, termed Yāvat Tāvat, were also explored.

In Astronomy, mathematics was intimately intertwined with astronomical studies. Ancient Indian thinkers initially developed mathematical knowledge as a part of astronomy. Aryabhata (born 476 CE) was a pivotal figure; his Aryabhatiya covers spherical trigonometry and astronomy. He proposed that the Earth is spherical and that the apparent motion of stars is due to the Earth's rotation. He also introduced the concept of sine (ardha-jya) and provided sine tables. Later, mathematicians like Madhava of Sangamagrama (around 1350 CE) and the Kerala School of Mathematics (14th-16th century CE) developed infinite series and calculus-

like ideas that enabled precise astronomical calculations, including for planetary motion and time calculations. The Sūryasiddhānta is another important astronomical text containing mathematical ideas.

(b) Trace the development of Indian mathematics from Vedic times to the time of Ramanujan. Mention key figures and their works.

The journey of Indian mathematics is long and rich, tracing back to Vedic times and culminating in the profound contributions of modern mathematicians like Ramanujan.

Vedic Times (around 1200 BCE onwards):The origins of Indian mathematics can be traced back to the Vedas. The Vedanga Jyotisa (around 1200 BCE) highlights the supreme importance of Ganitam (mathematics) and Jyotisam (astronomy). The **Sulba Sutras** (800-200 BCE) are among the earliest mathematical texts, found in Vedic literature. They contain geometrical rules for constructing altars, including the Pythagorean Theorem and approximations of irrational numbers like $\sqrt{2}$ and $\sqrt{3}$. Early concepts like the four fundamental operations (Parikarma), rule of three (Trairāśi), and basic algebra (Yāvat Tāvat) were explored.

Classical Period (300 BCE - 1200 CE):This period saw significant advancements:

Pingala (around 300 BCE):In his work Chandaḥśāstra, he explored Sanskrit prosody, using a binary number system and ideas related to combinatorics and the Fibonacci sequence. He also used the zero symbol (a dot).

Bakhshali Manuscript (200 CE): This ancient text on birch bark contains rules for arithmetic, fractions, square roots, and equations, and is one of the earliest known documents to use a symbol for zero, indicating advanced calculation methods.

Aryabhata (born 476 CE): A giant in Indian mathematics and astronomy. His famous book Aryabhatiya covers arithmetic, algebra, plane and spherical trigonometry, and astronomy. He explained the place-value system with zero, gave formulas for areas and volumes, the value of pi (accurate to four decimal places), and introduced the concept of sine. He also proposed a spherical Earth and its rotation.

Brahmagupta (598 CE): His major work, Brahmasphutasiddhanta, treated zero as a number and established rules for negative numbers, quadratic, and indeterminate equations. He also provided a formula for the area of a cyclic quadrilateral.

Mahavira (815-878 CE): A Jain mathematician who authored Ganita-Sara-Sangraha. He is credited with giving the general formula for binomial coefficients ($\binom{n}{r}=\frac{n!}{(n-r)!r!}$) for the first time.

Bhaskaracharya II (1114 CE): One of the greatest Indian mathematicians. His main work, Siddhanta Shiromani, includes Lilavati (arithmetic) and Bijaganita (algebra), covering equations, zero, and negative numbers. He also discussed early ideas of calculus, instantaneous motion, infinity, and developed the Chakravala method for solving Pell's equation.

Medieval Period (14th-16th Century CE):

Madhava of Sangamagrama (around 1350 CE): Founder of the Kerala School of Mathematics. He developed infinite series for π, sine, and cosine, foreshadowing calculus concepts in Europe.

Kerala School of Mathematics (14th-16th Century CE): This school, including mathematicians like Parameshvara, Nilakantha Somayaji, and Jyeshthadeva, further developed infinite series, calculus-like ideas, and precise astronomical calculations.

Modern Period (19th-20th Century CE):

Srinivasa Ramanujan (1887-1920): A self-taught genius who made extraordinary contributions to number theory, infinite series, and continued fractions. He discovered new formulas for π, profound results on partition functions, and introduced mock theta functions, significantly influencing later mathematics and physics. His work, supported by G. H. Hardy, gained international recognition and continues to inspire mathematicians.

This chronological overview highlights the continuous and evolving nature of mathematical inquiry in India, from its practical roots in Vedic rituals to its theoretical heights and global impact.