Indian Knowledge System – Mathematics

Chapter 1 : Introduction

1. Multiple Choice Questions

(a) Which of the following texts is considered the earliest source of Indian mathematics?

Ans: Sulba Sūtras

(b) Who among the following used a binary number system in prosody?

Ans: Pingala

(c) Which ancient Indian number name corresponds to 1012?

Ans: Parārdha

(d) In the Katapayādi system, the syllables “cha” and “na” correspond to which digits?

Ans: 6 and 0

2. True/False Questions

(a) Bhāskara II was the first to assign rules for working with zero.

False

(b) The Katapayādi system reads numbers from right to left in its later usage.

False

(c) The Bhūtasankhyā system uses meaningless syllables to represent numbers.

False

(d) The decimal place-value system and zero were developed in India by 5th century

True

3. Short Answer Questions

(a) What is the meaning of the verse ”Yathā śikhā mayūrāṇām...” and what does it signify?

Meaning: "Like the crests on the heads of peacocks, like the gems on the hoods of the cobras, mathematics (astronomy) is at the top of the Vedanga śāstras."

Significance: This verse from Vedanga Jyotisa signifies the supreme importance and esteemed position of mathematics and astronomy among the branches of secular knowledge in ancient India.

(b) Write any four Sanskrit number names for numbers from 1 to 9.

* Eka (एक) - 1

* Dvi (द्वि) - 2

* Tri (त्रि) - 3

(चतुः) - 4

(Other possible answers include Panca, Sat, Sapta, Asta, Nava)

(c) What is the main principle of the Bhūtasankhyā number system?

The main principle of the Bhūtasankhyā system is that numbers are represented using meaningful words related to objects, natural elements, or ideas, which are placed in the same order as digits in the decimal system. These words are often written and read from right to left.

(d) Mention two mathematicians from the Kerala School of Mathematics and their contributions.

Madhava of Sangamagrama (around 1350 CE): Known for starting the Kerala School of Mathematics and developing infinite series for pi, sine, and cosine, which laid the foundation for concepts that later became part of calculus.

Nilakantha Somayaji (15th-16th centuries CE): A mathematician from the Kerala School who improved earlier ideas and wrote commentaries on astronomy and mathematics, including explaining why Aryabhata called the value of pi 'asananh' or approximate (irrational).

(Other possible answers include Parameshvara and Jyeshthadeva.)

4. Medium Answer Questions

(a) Explain the role of the Sulba Sūtras in early Indian mathematics.

The Sulba Sutras, dating from around 800 CE - 200 CE, are among the earliest mathematical texts in India and are part of the Vedic literature. They played a crucial role by providing rules for constructing Vedic altars using ropes, thereby incorporating geometrical ideas. These texts clearly present the Pythagorean Theorem long before its known appearance in Greek mathematics. They also discuss methods for squaring the circle and provide approximate values for irrational numbers like $\sqrt{2}$ and $\sqrt{3}$, demonstrating the early development of mathematical thinking in India.

(b) Describe the structure and use of the Katapayādi system with one example.

The Katapayādi system is an ancient Indian method for representing numbers using letters, which facilitates memorization of large numbers by converting them into words or verses. In this system, specific consonants are assigned numerical values from 0 to 9, arranged into four groups starting with 'ka', 'ta', 'pa', and 'ya'. Vowels have no numerical value and are used solely to form pronounceable words. While early versions read numbers from right to left, later versions, like modern numbering, read from left to right. This system was particularly prevalent in South India for astronomy, mathematics, and poetry.

Example: The value of π to 17 decimal places is encoded in the verse: "भद्राम्बुधिगसिद्धजन्मगणितश्रद्धा स्म यद् भूपगीः"

The relevant consonants and their assigned digits are:

भ (4) द्रा (2) म्बु (3) धि (9) ग (7) सि (9) द्ध (8) ज (5) न्म (3) ग (5) णि (6) त (2) श्र (9) द्धा (5) स्म (1) य (4) द् (1) भू (3) प (1) गीः (3)

Reading these digits in reverse order gives the value of π as 3.14159265358979324.

(c) What is the significance of mathematics in the Indian tradition, according to ancient scholars?

According to ancient scholars, mathematics held a position of supreme importance and was considered both a practical and a pure subject. It was initially developed as part of astronomy and religious literature, and later evolved to understand the world, solve practical problems, and support disciplines like architecture, music, and logic. The verse from Vedanga Jyotisa, "Like the crests on the heads of peacocks, like the gems on the hoods of the cobras, mathematics (astronomy) is at the top of the Vedanga śāstras," highlights its elevated status. Indian mathematics developed independently, contributing concepts like zero and the decimal place-value system. It also fostered clear thinking, logic, and constructive proof techniques useful for algorithms. Furthermore, Indian mathematical texts were translated into Arabic and Latin, facilitating the transmission of these ideas globally. Mahavira emphasized that mathematics is useful and necessary in all areas of life, from daily matters and Vedic rituals to religious duties, love, trade, music, and drama.

5. Long Answer Questions

(a) Discuss the major contributions of ancient Indian mathematicians to arithmetic, algebra, and astronomy with appropriate examples.

Ancient Indian mathematicians made remarkable contributions to arithmetic, algebra, and astronomy.

In Arithmetic, one of the greatest achievements was the invention of the decimal place-value system and the use of zero as a number. This foundationally changed counting and calculation methods worldwide. The concept of zero was treated as a number and used in calculations by Brahmagupta (598 CE). Basic operations like addition, subtraction, multiplication, and division (Parikarma) were well-established. The "rule of three" (Trairāśi) was also developed for practical computation. Early texts like the Bakhshali manuscript (200 CE) show advanced calculation methods, including those for fractions and square roots. Śrīdhara (around 750 CE) and Bhaskara II (1150 CE) further elaborated on number names and place value.

For Algebra, India's contributions were foundational, particularly the development of Avyaktaganita (algebra). Brahmagupta (598 CE) gave rules for working with negative numbers and methods to solve quadratic and indeterminate equations, which are still taught today. The Kuttaka method was developed for solving indeterminate equations. Bhaskara II (1114 CE) further advanced algebraic problems in his work Bijaganita, building upon the use of zero and negative numbers. He is also known for developing the Chakravala method, a powerful technique to solve Pell's equation ($Nx^2 + 1 = y^2$), a significant achievement in number theory. Simple algebraic equations, termed Yāvat Tāvat, were also explored.

In Astronomy, mathematics was intimately intertwined with astronomical studies. Ancient Indian thinkers initially developed mathematical knowledge as a part of astronomy. Aryabhata (born 476 CE) was a pivotal figure; his Aryabhatiya covers spherical trigonometry and astronomy. He proposed that the Earth is spherical and that the apparent motion of stars is due to the Earth's rotation. He also introduced the concept of sine (ardha-jya) and provided sine tables. Later, mathematicians like Madhava of Sangamagrama (around 1350 CE) and the Kerala School of Mathematics (14th-16th century CE) developed infinite series and calculus-

like ideas that enabled precise astronomical calculations, including for planetary motion and time calculations. The Sūryasiddhānta is another important astronomical text containing mathematical ideas.

(b) Trace the development of Indian mathematics from Vedic times to the time of Ramanujan. Mention key figures and their works.

The journey of Indian mathematics is long and rich, tracing back to Vedic times and culminating in the profound contributions of modern mathematicians like Ramanujan.

Vedic Times (around 1200 BCE onwards):The origins of Indian mathematics can be traced back to the Vedas. The Vedanga Jyotisa (around 1200 BCE) highlights the supreme importance of Ganitam (mathematics) and Jyotisam (astronomy). The **Sulba Sutras** (800-200 BCE) are among the earliest mathematical texts, found in Vedic literature. They contain geometrical rules for constructing altars, including the Pythagorean Theorem and approximations of irrational numbers like $\sqrt{2}$ and $\sqrt{3}$. Early concepts like the four fundamental operations (Parikarma), rule of three (Trairāśi), and basic algebra (Yāvat Tāvat) were explored.

Classical Period (300 BCE - 1200 CE):This period saw significant advancements:

Pingala (around 300 BCE):In his work Chandaḥśāstra, he explored Sanskrit prosody, using a binary number system and ideas related to combinatorics and the Fibonacci sequence. He also used the zero symbol (a dot).

Bakhshali Manuscript (200 CE): This ancient text on birch bark contains rules for arithmetic, fractions, square roots, and equations, and is one of the earliest known documents to use a symbol for zero, indicating advanced calculation methods.

Aryabhata (born 476 CE): A giant in Indian mathematics and astronomy. His famous book Aryabhatiya covers arithmetic, algebra, plane and spherical trigonometry, and astronomy. He explained the place-value system with zero, gave formulas for areas and volumes, the value of pi (accurate to four decimal places), and introduced the concept of sine. He also proposed a spherical Earth and its rotation.

Brahmagupta (598 CE): His major work, Brahmasphutasiddhanta, treated zero as a number and established rules for negative numbers, quadratic, and indeterminate equations. He also provided a formula for the area of a cyclic quadrilateral.

Mahavira (815-878 CE): A Jain mathematician who authored Ganita-Sara-Sangraha. He is credited with giving the general formula for binomial coefficients ($\binom{n}{r}=\frac{n!}{(n-r)!r!}$) for the first time.

Bhaskaracharya II (1114 CE): One of the greatest Indian mathematicians. His main work, Siddhanta Shiromani, includes Lilavati (arithmetic) and Bijaganita (algebra), covering equations, zero, and negative numbers. He also discussed early ideas of calculus, instantaneous motion, infinity, and developed the Chakravala method for solving Pell's equation.

Medieval Period (14th-16th Century CE):

Madhava of Sangamagrama (around 1350 CE): Founder of the Kerala School of Mathematics. He developed infinite series for π, sine, and cosine, foreshadowing calculus concepts in Europe.

Kerala School of Mathematics (14th-16th Century CE): This school, including mathematicians like Parameshvara, Nilakantha Somayaji, and Jyeshthadeva, further developed infinite series, calculus-like ideas, and precise astronomical calculations.

Modern Period (19th-20th Century CE):

Srinivasa Ramanujan (1887-1920): A self-taught genius who made extraordinary contributions to number theory, infinite series, and continued fractions. He discovered new formulas for π, profound results on partition functions, and introduced mock theta functions, significantly influencing later mathematics and physics. His work, supported by G. H. Hardy, gained international recognition and continues to inspire mathematicians.

This chronological overview highlights the continuous and evolving nature of mathematical inquiry in India, from its practical roots in Vedic rituals to its theoretical heights and global impact.

Chapter 2 Contributions to Geometry

1. Multiple Choice Questions (MCQ).

(a) (C) 9

(b) (C) Baudhāyana Śulbasūtra

(c) (C) Brahmagupta

(d) (D) Constructing cyclic quadrilaterals

2. True/False Questions

(a) True (b) True (c) False (d) False

3. Short Answer Questions:

(a) Name the three sacred fires mentioned in the Śulbasūtras ?

Ans: The three sacred fires mentioned in the are: Daksinagni, Garhapatya, Ahavaniya

(b) Find two sides of a right-angled triangle if hypotenuse is 15.

Ans: $a^2 + b^2 = 15^2 \rightarrow a^2 + b^2 = 225$

This gives $a = 9, b = 12$

(c) Using Mahavira's formula, find the circumference of an ellipse with semi-major axis 5 and semi-minor axis 4.

Ans: Use the formula $C = \sqrt{16a^2 + 24b^2}$ with $a = 5, b = 4$

This gives $C = 28$

4. Medium Answer Questions:

(a) Given two sides of two right-angled triangles say (9, 12, 15) and (12, 5, 13), can you construct a cyclic quadrilateral with rational sides ? If yes, find the diagonals of such quadrilateral.

Ans: **Yes**, a cyclic quadrilateral with rational sides can be constructed using Brahmagupta's method, which takes two right-angled triangles with rational sides.

For simplicity, let's use the primitive triple of the first set: $(3,4,5)$

Let $a = 3, b = 4, c = 5$ and $x = 12, y = 5, z = 13$

The sides of the resulting quadrilateral are proportional to cy, az, cx, bz

$$az = 3 \times 13 = 39$$

$$bz = 4 \times 13 = 52$$

$$cx = 5 \times 12 = 60$$

$$cy = 5 \times 5 = 25$$

The sides are $39,52,60,25$

Let $a = 39, b = 52, c = 60, d = 25$

Brahmagupta's formula for the diagonals x and y of a cyclic quadrilateral with sides a, b, c, d is:

$x = \sqrt{\frac{(ab+cd)(ac+bd)}{ad+bc}}$ and $y = \sqrt{\frac{(ad+bc)(ac+bd)}{ab+cd}}$

This gives $x = 56, y = 65$

The diagonals of the cyclic quadrilateral are **56** and **65**.

(b) Can you list some of the earliest known instances where the Pythagorean Theorem was used or mentioned, long before Pythagoras lived?

Ans: **The Sulbasutras (Vedic/Sulbasutra Period)**: These texts, composed around the eighth or ninth century BCE , state the theorem (Bhuja Koti Karani Nyayam). It is described as: "The diagonal of a rectangle (or the hypotenuse of a right-angled triangle) is such that the square on it is equal to the sum of the squares on the two sides". Geometry in this period was used to build altars for religious purposes.

(c) Explain briefly how the Śulbasūtras were important in the cultural and religious practices of the Vedic period.

Ans: The Śulbasūtras were fundamentally important to the religious practices of the period because they provided the exact geometrical rules for building sacred fire altars and fireplaces .

- **Foundation of Rituals**: Vedic rituals were an important part of life around the time the Śulbasūtras were composed. The religious efficacy of these rituals depended on the altars being constructed with **specific shapes and exact sizes**.
- **Geometric Rules**: The texts describe step-by-step rules for constructing shapes like squares, rectangles, circles, and semicircles using ropes .
- **Development of Geometry**: The necessity for this precision in religious construction led directly to the early development of mathematics and geometry in India. The Śulbasūtras contain rules for building altars with fixed areas, which included early geometric reasoning and methods to approximate values like $\sqrt{2}$.

5. Long Answer Questions:

(a) State and prove the Brahmagupta's formula for the Area of a cyclic quadrilateral in terms of sides.

Ans: Refer textbook page number 29 (2.4.2)

(b) Give Baudhayana's method for circling a square.

Ans: **Construction Method**

1. **Start with the Square**: Consider the square $ABCD$.
2. **Find Center and Midpoint**: Let O be the center of the square (intersection of diagonals). Let M be the midpoint of side CD.
3. **Draw Arc and Intersection**: Draw a circle with center O and radius OD (half the diagonal). Let this circle intersect the line segment OM(extended to the outside of the square) at point N.
4. **Mark Point** : Divide the segment MN at point P such that $PM = \frac{1}{3}MN$.
5. **Construct the Circle**: The new circle is constructed with center O and radius OP .

The area of this resulting circle with radius OP is approximately equal to the area of the original square ABCD.

(c) Give Ramanujan's method (with proof) for constructing a square whose area is nearly equal to that of a given circle.

Ans: Refer textbook page number 26 (2.3.2)

Chapter 3
Contributions to Arithmetic and Number Theory

1. **Multiple Choice Questions**.

(a) (C) Bakhshali Manuscript

(b) (A) Aryabhata

(c) (B) Pell's equation

2. **State True or False.**

(a) False

(b) True

(c) True

(d) False

3. Short Answer Questions.

(a) Mention two key features of the Indian numeral system.

Ans: The use of **place value** based on powers of ten. The invention of **zero** both as a symbol and as a number.

(b) Write one difference between "abacists" and "algorists".

Ans: **Abacists** were those who performed calculations in Europe primarily using the **abacus** and **Roman numerals**. **Algorists** were those who used the **new Indian decimal place-value system** (which simplified calculations). The term "algorism" later evolved into the modern word "**algorithm**".

(c) Who translated Brahmagupta's works into Arabic in the 8th century?

Ans: **Al-Fazari** translated Brahmagupta's works (*Brahmasphutasiddhānta* and *Khandakhādyaka*) into Arabic in the 8th century, which helped introduce Indian numerals to Arab scholars.

(d) Using an A.P., show that $\boldsymbol{n^3 = n + 3n + 5n + \cdots + (2n-1)n}$

Ans: Here $d = 2n, a = n, l = 2n^2 - n$ and a number of terms $N = n$

The formula for the sum of an A.P. is $S = \frac{N}{2}(a + l)$

$$S = \frac{n}{2}(n + (2n^2 - n))$$

$$S = n^3$$

4. Medium Answer Questions.

(a) Find all positive integer solutions (x, y) to the equation $\frac{11x+94}{3} = y$

Ans: $3y - 11x = 94$

$$3y = 11x + 94$$

$$y = \frac{11x + 94}{3} = 3x + 31 + \frac{2x + 1}{3}$$

Let $k = \frac{2x+1}{3} \rightarrow x = \frac{3k-1}{2} = k + \frac{k-1}{2}$

Let $m = \frac{k-1}{2} \rightarrow k = 2m + 1$

$$x = k + m = (2m + 1) + m = 3m + 1$$

$$3y = 11(3m + 1) + 94$$

$$y = 11m + 35$$

Smallest Positive Solution (when $m = 0$):

$$x_0 = 3(0) + 1 = 1$$

$$y_0 = 11(0) + 35 = 35$$

The general solution for all positive integer pairs (x, y) is:

$$x = 1 + 3t, y = 35 + 11t$$

(b) Find all positive integer solutions (x, y) to the equation $\frac{27x+5}{8} = y$

Ans: The equation can be rewritten as $8y - 27x = 5$

This gives $y = \frac{27x+5}{8} = 3x + \frac{3x+5}{8}$

Let $\frac{3x+5}{8} = k$ where k is an integer.

$$3x + 5 = 8k \rightarrow 3x = 8k - 5$$

$$x = \frac{8k-5}{3} = (2k-1) + \frac{2k-2}{3} = (2k-1) + \frac{2(k-1)}{3}$$

Let $\frac{k-1}{3} = m$ where m is an integer.

$$k - 1 = 3m \rightarrow k = 3m + 1$$

Smallest Positive Solution (when $m = 0$):

$$k = 3(0) + 1 = 1$$

$$x_0 = \frac{8(1)-5}{3} = 1$$

$$y_0 = \frac{27(1)+5}{8} = 4$$

All positive integer solutions: $x = x_0 + bt$ and $y = y_0 + at$

$$x = 1 + 8t, y = 4 + 27t$$

(c) A person pays for a horse in the following way: 1 coin on the first day, 3 on the second, 5 on the third, and so on, increasing the payment by 2 coins each day. In how many days will the full price of 100 coins be paid?

Ans: Here $a = 1, d = 2, S = 100$

$$S = \frac{n}{2}[2a + (n-1)d]$$

This gives $n = 10$

The full price will be paid in **10 days**.

(d) Explain briefly the role of Aryabhata in the development of the place-value system without zero.

Ans: Aryabhata (499 CE) played a key role by describing a decimal **positional system** in his *Āryabhatīya*. This system used **Sanskrit letters** to represent numbers. Crucially, for an empty place, he used the word **'kha'** (ख) meaning "space" or "emptiness". While 'kha' indicated the absence of a digit, it was considered a **placeholder** and not yet formally treated as a **number**. This established the principle of position-based value before the official invention of zero as a number by later mathematicians like Brahmagupta.

(e) Explain briefly how Indian numeral system propagated to Europe.

Ans: The Indian decimal number system and the concept of zero spread from **India to the Arab world** and subsequently to Europe.

1. **To the Arab World:** In the 8th century, **Al-Fazari** translated Brahmagupta's works (*Brahmasphutasiddhānta* and *Khandakhādyaka*) into Arabic, thereby introducing Indian numerals to Arab scholars.

2. **To Europe:** The Arab mathematician **'Al-Khwarizmi'** (9th century) wrote a book on the Indian calculation method, which was later translated into Latin (*Liber Algoritmi de numero Indorum*). The system was finally popularized across Europe by **Fibonacci's book *Liber Abaci*** in 1202 CE.

5. Long Answer Questions.

(a) Find the smallest positive integer *N* such that

***N* ≡ 5 (mod 8) and *N* ≡ 2 (mod 27).**

Ans: $8x + 5 = 27y + 2 \rightarrow 8x - 27y = -3$

We use the Kuttaka principle (algebraic solution):

$$8x = 27y - 3 \rightarrow x = \frac{27y - 3}{8} = 3y + \frac{3y - 3}{8} = 3y + \frac{3(y - 1)}{8}$$

Let $\frac{y-1}{8} = k \rightarrow y = 8k + 1$

We seek the smallest positive integer N , which corresponds to the smallest non-negative integer k.

Let $k = 0$

$$y = 8(0) + 1 = 1$$

$$N = 27y + 2 = 27(1) + 2 = 29$$

The smallest positive integer N is **29**.

Chapter 4 : Algebra and Calculus

1. **Multiple Choice Questions**
   (a) B) Algebra
   (b) B) Brahmagupta
   (c) C)
   (d) B) Jyesthadeva
   (e) B) Infinite series, early calculus, and astronomy
2. **True / False Questions.**
   (a) True
   (b) False
   (c) True
   (d) True
   (e) False
3. **Short Answer Questions:**

(a) Differentiate between Bijaganita and Vyaktaganita as explained by ancient Indian mathematicians.

Ans: **Bijaganita** (बीजगणित) is the term for **Algebra**. The word *bija* means "seed," and *ganita* means "the science of calculation". It is also known as **avyakta-ganita** (अव्यक्तगणित), meaning the **"calculation with unknowns/unmanifested"**. It is considered the **source** of arithmetic. **Vyaktaganita** (व्यक्तगणित) is the term for **Arithmetic**. It means **"calculation with knowns/manifested"**.

(b) Write a short note on Aryabhata's approximation of π and compare it with the Jaina value of π.

Ans: **Aryabhata's approximation of π** was a significant breakthrough in ancient mathematics, providing the most accurate value known worldwide in his time. Around 499 CE, Aryabhata calculated π as $\frac{62832}{20000}$, which simplifies to **3.1416**. This value was **correct to four decimal places** and was explicitly stated by him to be an **approximation** (*āsanṇa*), demonstrating an understanding of the irrational nature of the number. In contrast, the **Jaina value of π** , found in Jaina texts dating centuries earlier, was approximated as $\sqrt{10}$, which is approximately **3.162**. While the Jaina value was used by several mathematicians

including Brahmagupta, it was **less accurate** than Aryabhata's, with an error of about 0.7%, highlighting the substantial refinement in calculation achieved by Aryabhata.

(c) Name any two Mathematicians from Kerala school and their texts.

Ans:

 Madhava of Sangamagrama (1340-1425 CE). He founded the Kerala School and discovered infinite series for trigonometric functions and π .

 Jyesthadeva (c. 1500–1575 CE). He authored the groundbreaking book **Yuktibhasha**, which systematically explained the calculus concepts of the Kerala School.

 Nilakantha Somayaji (1444–1544 CE). He authored the book **Tantrasangraha** and refined infinite series for π , sine, and cosine.

 Parameshvara (c. 1360–1455 CE). He developed the **Drgganita** system.

4. Medium Answer Questions:

(a) Explain the contribution of Brahmagupta in solving quadratic equations.

Ans: For the quadratic equation $ax^2 + bx = c$, Brahmagupta gave the solution for x as:

$$x = \frac{\sqrt{4ac + b^2} - b}{2a}$$

This formula is exactly the same as the modern quadratic formula when considering only the positive root.

The modern formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The term $4ac + b^2$ is equivalent to the discriminant

$b^2 - 4a(-c) = b^2 + 4ac$

from the standard form $ax^2 + bx - c = 0$

Brahmagupta's formula for (the number of terms) in a quadratic equation derived from an arithmetic progression was also given by Sridhara and is found in the Bakhshali Manuscript.

(b) Explain in brief the contribution of Bhaskara II to the early development of calculus.

Ans: Bhaskara II (1150 CE) made several key contributions that indicate an early understanding of calculus concepts:

- **Derivative of the Sine function:** Bhaskara II understood that the derivative of the sine function is the cosine function. This is clear from a verse in his *Goladhyaya*, which in modern language is expressed as: $\frac{d}{d\theta}(\sin\theta) = \cos\theta$
- **Concept of Zero Derivative at Maximum (Maximization Principle):** He articulated a principle that is fundamental to differential calculus: **"Where the planet has its maximum equation (*phala*), there the *gatiphala* (result of motion) must be absent (zero)."**. In modern terms, this means that a function $f(t)$ reaches its maximum value where its derivative $f'(t)$ is zero.
- **Volume of a Sphere (Early Integration):** He provided the correct formulas for the surface area and volume of a sphere. His method for finding the volume involved conceptually dividing the sphere's surface into **infinitely small pieces**, forming small pyramids, and adding their volumes, a concept similar to integration.

(c) Write a note on contributions of the Kerala School of Mathematics.

Ans: The Kerala School of Mathematics, founded by **Madhava of Sangamagrama** (1340-1425 CE), was a major center of mathematical innovation that lasted for about 250 years.

- **Foundational Calculus Concepts:** Their most significant work was in areas related to calculus, including **differentiation, integration, and infinite series**. These ideas were developed long before they were formally created by Newton and Leibniz in Europe.

- **Infinite Series:** Madhava discovered **infinite series expansions** for trigonometric functions (sine, cosine, and arc tangent). This included the famous series for $\frac{\pi}{4}$.
- **Numerical Accuracy:** Using these series, Madhava calculated the value of π **accurately up to 11 decimal places**.
- **Systematic Documentation:** The work of the school was systematically compiled and explained in texts like the **Yuktibhasha** by **Jyesthadeva**, which is considered by many scholars to be the world's first book on calculus.
- **Astronomy:** The school also made significant contributions to astronomy, with figures like **Nilakantha Somayaji** proposing a partially heliocentric model and **Parameshvara** developing the *Drgganita* system.

(d) In a forest, there are four herds of elephants: elephants in rut (matta), elephants not in rut (amatta), female elephants and young elephants. If we exclude one herd at a time, the sum of elephants in the remaining three herds is found to be 30, 36, 49 and 50 respectively. Find the total number of elephants in the forest and also determine the number of elephants in each herd.

Ans: Let x_1, x_2, x_3, x_4 be the number of elephants in the four herds (matta, amatta, female, young).

Let S be the total number of elephants: $S = x_1 + x_2 + x_3 + x_4$

The sums when one herd is excluded are given by the system of linear equations:

$$S - x_1 = 30, S - x_2 = 36, S - x_3 = 49, S - x_4 = 50$$

$$S - x_1 + S - x_2 + S - x_3 + S - x_4 = 165$$

$$4S - S = 165$$

$$S = 55$$

The **total number of elephants** in the forest is **55.**

The number of elephants in the four herds are **25, 19, 6, and 5**.

5. Long Answer Questions:

(a) Explain in detail, how the value of π has evolved with time in Ancient Indian Mathematics.

Ans: The idea of π appeared in problems like "squaring the circle". In the **Sulbasutras** (800-500 BCE), π was approximated as **3.088**. Jaina texts (c. 5th century BCE - 5th century CE) suggested the approximation: $\pi \approx \sqrt{10} = 3.162$. This value, with an error of less than 0.7%, was widely used by mathematicians like **Brahmagupta** and continued to be used in India until the 15^{th} century. **Aryabhata** was the first Indian mathematician to provide a highly accurate value: $\pi = \frac{62832}{20000} = 3.1416$. This value was **correct to four decimal places** and was the best approximation known globally at the time. Bhaskara I was used the fraction $\frac{22}{7}$ as a simple and very good fraction for π. **Bhaskara II** gave the same value as Aryabhata, $\pi = \frac{3927}{1250}$ =**3.1416**. **Madhava** of the Kerala School discovered an **infinite series** for π , later known in Europe as the Gregory-Leibniz series. Using this series, he was able to calculate π with an accuracy of **11 decimal places**. **Ramanujan** derived several rapidly converging series for $\frac{1}{\pi}$.

6. A young woman's pearl necklace broke during a quarrel, scattering all of its pearls. Onethird of the pearls fell onto the floor, and one-fifth landed on the bed. The woman herself gathered one-sixth of the pearls, while her lover collected one-tenth. They noticed that six pearls remained hanging from the broken thread. How many pearls were in the necklace originally?

Ans: Let x be the original number of pearls in the necklace. The fractions of pearls that fell are $\frac{x}{3}, \frac{x}{5}, \frac{x}{6}, \frac{x}{10}$

The sum of all parts must equal the total number of pearls, x:

$$x = \frac{x}{3} + \frac{x}{5} + \frac{x}{6} + \frac{x}{10} + 6$$

$$x = \frac{24}{30}x + 6$$

$$x = 30$$

The number of pearls in the necklace originally was **30**.

7. **A herd of elephants was seen in two different places. On a mountain slope, observers saw one-third of the entire herd and three times the square root of the remaining elephants. In a lake, they saw a male elephant along with three female elephants. How many elephants were there in the total herd?**

   Ans:

Let x be the total number of elephants in the herd. The observers saw elephants in two places: on a mountain slope and in a lake.

1. Elephants on the mountain slope:

   Slope $= \frac{1}{3}x + 3\sqrt{(x - \frac{1}{3}x)}$

2. **Elephants in the lake:** $1 + 3 = 4$

   The sum of the elephants seen in both places must equal the total herd x:

$$x = \frac{1}{3}x + 3\sqrt{\frac{2}{3}x} + 4$$

$$x - \frac{1}{3}x - 4 = 3\sqrt{\frac{2}{3}x}$$

$$x = 24$$

The total number of elephants in the herd was **24**.

8. **During the battle, Arjuna shot a certain number of arrows at Karna.With half of his arrows, he destroyed Karna's arrows. Then,**

with four times the square root of the total arrows, he killed Karna's horses. After that, he hit Shalya with six arrows and destroyed Karna's umbrella, flag, and bow using three arrows. Finally, he beheaded Karna with one arrow. How many arrows did Arjuna shoot in total?

Ans: Let x be the total number of arrows Arjuna shot.

$$x = \frac{1}{2}x + 4\sqrt{x} + 6 + 3 + 1$$

$$x - \frac{1}{2}x - 10 = 4\sqrt{x}$$

$$x = 100$$

Arjuna shot a total of **100** arrows.

9. **At the beginning of the rainy season, ten times the square root of the total number of swans flew away to Manasa Sarovara, and one-eighth of the swans went to the Sthalapadminee forest. Three pairs of swans were seen playing in the lotus-filled water. Find the total number of swans in the group.**

Ans: Let x be the total number of swans in the group.

$$x = 10\sqrt{x} + \frac{1}{8}x + 6$$

$$\frac{7}{8}x - 6 = 10\sqrt{x}$$

$$x = 144$$

The total number of swans in the group was **144**.

Chapter 5

Combinatorics and Astronomy-Related Mathematics

1. **Multiple Choice Questions (MCQ):**
   (a) (B) Pascal's triangle
   (b) (B) 2^n
   (c) (B) Mahavira
   (d) (C) Large spans of time
2. **State True or False:**
   (a) True
   (b) False
   (c) True
   (d) False
3. **Short Answer Questions:**
   (a) Write the recurrence relation for binomial coefficients given by Pingala.
   Ans: ${}^{n}C_{r}={}^{n-1}C_{r-1}+{}^{n-1}C_{r}$
   (b) What is the significance of the Adhika Masa in the Indian calendar system?
   Ans: The **Adhika Masa** or extra month, is occasionally added to the luni-solar calendar system to keep the calendar in balance, since lunar months are shorter than solar months.
   (c) Name two astronomers of the Kerala School and one of their mathematical contributions.
   Ans: One notable astronomer of the Kerala School is **Madhava**. A mathematical contribution of the school was producing **highly precise values of π** and improving Aryabhata's sine values correct up to 7 decimal places.

(d) State Bhaskara I's rational approximation formula for sin θ.

Ans: $\sin\theta = \frac{4(180^0-\theta)\theta}{40500-(180^0-\theta)\theta}$

4. Medium Answer Questions:

(a) Explain briefly how Sushruta used combinations to calculate the number of possible tastes from six basic tastes.

Ans: Sushruta calculated the number of different tastes that can be made by combining the **six basic tastes** by summing the combinations of these tastes taken one at a time, two at a time, and so on, up to all six at once.

$$6C1 + 6C2 + 6C3 + 6C4 + 6C5 + 6C6$$
$$= 6 + 15 + 20 + 15 + 6 + 1 = 63$$

(b) What is the relation between Pingala's Meru-prastara and the binomial theorem?

Ans: Pingala's **Meru-prastara** is a triangular arrangement of numbers that is identical to what is known in the West as Pascal's triangle. These numbers are the **binomial coefficients nCr.** The binomial coefficients are the coefficients in the **binomial expansion** of $(a+b)^n$, which is the Binomial Theorem.

(c) Describe how the Yuga system was used to express large spans of time.

Ans: The Yuga system is based on a **Mahayuga**, which is a cycle composed of four smaller Yugas (Krutyuga, Tretayuga, Dwaparyuga, and Kaliyuga). The duration of one Mahayuga is the sum of the human years of the four Yugas, totaling **4,320,000 solar years**. Ancient texts further describe the time span of Brahma, where **1 day of Brahma** (a **Kalpa**) is equal to **1,000 Mahayugas**, which is 4.32×10^9 solar years. This system allows for the calculation of the immense lifespan of Brahma (a **Mahakalpa**) which is about **311 trillion and 40 billion human years**.

(d) Describe the significance of the city of Ujjain (Avanti) and the mathematical reference point 'Lanka' in ancient Indian astronomy.

Ans: The city of Ujjain was highly significant because it was defined as the location for the ancient Indian **prime meridian** (0^0 longitude) used for astronomical calculations. The city, symbolized by the Mahakaleshwar Temple, was a historic center for timekeeping and astronomy. **Lanka** was a **mathematical reference point** defined by astronomers like Aryabhata. It was the point where the prime meridian passing through Ujjain crossed the equator. This 'Lanka' is not the island of Sri Lanka but a theoretical location, used as a reference for time-zone calculations, such as in Varahamihira's description of four cities where different times (sunrise, noon, sunset, midnight) occur simultaneously.

5. Long Answer Questions:

(a) Discuss Pingala's contributions to combinatorics, focusing on his work on poetic meters, Meru-prastara, and binary numbers. Give examples.

Ans: Pingala, who authored the *Chhandasutra* (around 200 BCE), made foundational contributions to combinatorics, primarily through his study of **Sanskrit prosody** (poetic meters). Pingala studied how many different patterns of verses can be formed with a given number of syllables, where each syllable is either **laghu (l)** (short, 1 matra) or **guru (g)** (long, 2 matras). He established the basic combinatorial rule that for n syllables, the total number of possible patterns is 2^n . For example, with two syllables, there are patterns (). He also posed the central combinatorial problem of finding the number of distinct rhythms of n syllables that contain exactly r laghu syllables and $n - r$ guru syllables. The solution is the number of combinations, nCr .

Pingala provided a formula for the binomial coefficients, which, in modern notation, is the recurrence relation: ${}^{n}C_{r}={}^{n-1}C_{r-1}+{}^{n-1}C_{r}$. He

arranged these coefficients in a triangular form called the **Meru-prastara** , which is exactly the same as the structure later known as Pascal's triangle in the West. This triangle shows the coefficients for the binomial expansion of $(a + b)^n$.

Pingala described an algorithm to generate all possible meters for a fixed number of syllables. Later commentators like Halayudha demonstrated that if a guru syllable is assigned a **0** and a laghu syllable is assigned a **1**, reversing the sequence yields the **binary number** corresponding to the meter's index minus one . This algorithm, which he also used in the process called *nastam* to recover a meter from its index, provided the world's **first known method to represent binary numbers**.

(b) Write an account of Aryabhata's sine table: why he chose R = 3438, how accurate his values were, and how it connects with modern trigonometry.

Ans: Aryabhata defined the sine not as a ratio, but as the length of half a chord in a circle of a specific radius R. The value was chosen based on his value for π and the total arc minutes in a full circle . The radius was calculated such that the circumference is $2\pi R$.

$$\frac{circumference}{2\pi} = \frac{360 \times 60'}{2 \times 3.1416} \approx 3438$$

Aryabhata's table provided 24 values for $R \sin\theta$. Comparing the values in the table, it is evident that his calculated sine values were **extremely close** to the actual values $(3438 \times \sin\theta)$ for those specific angles.

The **discrete version of the second derivative**, recognizing the proportionality:

$$\frac{d^2(\sin\theta)}{d\theta^2} \propto \sin\theta$$

The iterative summation of differences in his algorithm mirrors the integral operation, suggesting an implicit use of the **Fundamental Theorem of Calculus**.

Chapter 6 : Legacy of Indian Mathematics and Vedic Techniques

1. **Choose the correct option:**
   (a) (B) Bijaganita
   (b) (B) Method of exhaustion
   (c) (B) Cardinalities of Sets (Cantor's theory)
   (d) (A) Multiplication
   (e) (C) Logarithms
2. **State True or False:**
   (a) False
   (b) False
   (c) False
   (d) True
   (e) False
3. **Short answer questions:**
   (a) Name any two proof techniques employed by Indian mathematicians from the Kerala School.
   Ans:**Method of Exhaustion** (an early idea of calculus)
   Iterative and Recursive Methods (used by Madhava for the infinite series for $\frac{\pi}{4}$)
   (b) What is the meaning of the Sanskrit term Upapatti as emphasized by Bhaskara II?
   Ans: *Upapatti* means **demonstration** or **proof/justification**, which Bhaskara II stressed the importance of in his work *Goladhyaya*.
   (c) According to Jaina classification, what is the key difference between an Asamkhyeya number and an Ananta number?
   Ans: **Asamkhyeya** (असंख्येय) numbers are **uncountable numbers** that are larger than countable numbers but **cannot be listed**. **Ananta** (अनन्त) numbers are **infinite numbers** that are **endless** and larger than uncountable numbers.

(d) Define the Jaina classification of numbers: Samkhyeya, Asamkhyeya, and Ananta.

Ans: **Samkhyeya** (संख्येय): **Countable numbers**, which can be enumerated.

Asamkhyeya (असंख्येय): **Uncountable numbers**, larger than countable, that cannot be listed.

Ananta (अनन्त): **Infinite numbers**, endless and larger than uncountable.

(e) Name any 5 Indian Mathematicians and their major works (texts).

Ans: **Aryabhata** (476–550 CE): *Aryabhatiya*

Brahmagupta (598–668 CE): *Brahmasphutasiddhanta*

Bhaskara II (Bhaskaracharya) (1114–1185 CE): *Lilavati* (arithmetic, geometry), *Bijaganita* (algebra), *Siddhanta Siromani* (astronomy)

Mahavira (9th century CE): *Ganita Sarasangraha*

Pingala (c. 300 BCE): *Chhandasutra*

(f) Find 888 × 991 by Urdhva Tiryagbhyam Sutra.

Ans: $888: a = 8, b = 8, c = 8$

$991: d = 9, e = 9, f = 1$

Step	Operation	Result	Carry (from right)
Vertical right ($c \times f$)	$8 \times 1 = 8$	8	0
Crosswise right ($bf + ce$)	80	0	8
Crosswise full and middle vertical ($af + be + cd$)	152	$152 + 8 = 160$	16
Crosswise left ($ae + bd$)	144	$144 + 16 = 160$	16
Vertical left ($a \times d$)	72	$72 + 16 = 88$	0

The final product, reading the unit digits (and the final result) from left to right: **880008**

(g) Find $\frac{212}{9}$ by Nikhilam method.

Split the dividend (212) into quotient and remainder parts. Since the divisor (9) is a single digit, the remainder part is one digit: $21 \mid 2$.

Bring down the first digit: The first quotient digit is 2.

Add and carry:

a. Next dividend digit is 1. Add it to the previous quotient digit: $1 + 2 = 3$. This is the next quotient digit. $23 \mid 2$.
b. Next dividend digit is 2 (in the remainder part). Add it to the last quotient digit: $2 + 3 = 5$. This is the final remainder.

Result: **Quotient is 23, Remainder is 5**.

4. Medium answer questions:

(a) Write a short note on proof techniques by Indian Mathematicians.

Ancient Indian mathematicians, particularly those from the **Kerala School** (c. 14th-16th centuries), developed rigorous and sophisticated methods to verify their mathematical results, contrary to an old misconception that they only stated rules without proof. Their style was different from later European axiomatic-deductive approaches but was equally logical.

Key techniques included:

(a) **Step-by-Step Derivation**: A general method was illustrated by solving a specific example and clearly explaining the logic behind each step, showing its universal applicability.

(b) **Visual and Geometric Proofs**: Algebraic results were often established through **geometric algebraic arguments**, relying on precise Sanskrit descriptions of shapes, areas, and transformations.
(c) **Method of Exhaustion**: This technique, an early idea of **calculus**, was used to approximate values like . It involved calculating the area of a circle by inscribing polygons with an increasing number of sides, demonstrating an early understanding of **limits** and **infinity**.
(d) **Proof via Commentary (*Tika*)**: Later scholars wrote detailed commentaries on foundational texts, not only explaining but also deriving and expanding upon earlier results, providing essential justifications (*upapatti*).

(b) Discuss the role of Buddhist monasteries like Nalanda in the development and dissemination of Indian mathematics and astronomy.

Ans: Buddhist monasteries such as **Nalanda** and **Vikramasila** were vital **centers of learning** that significantly contributed to the development and spread of Indian mathematics and astronomy.

- **Development and Practice**: Monks within these centers studied **astronomy** rigorously, primarily for practical purposes such as preparing **calendars** and accurately fixing **ritual dates**. This necessary application of astronomical knowledge required a strong foundation in mathematics, including concepts like the **place value system** and **large-number arithmetic**, which were often inspired by cosmological texts like the *Lokaprajnapti*.
- **Dissemination**: Buddhist teachers and scholars acted as conduits, carrying Indian mathematical and astronomical knowledge to other countries. Figures like **Dharmapala** and translators such as **Xuanzang** and **Yijing** were instrumental in spreading these ideas to **China** and further into **East Asia**. This exchange incorporated Indian methods of calculation, calendar-making, and astronomical observations into the wider Buddhist world.